Purpose: In this problem set, you will develop an intuitive understanding of inverse functions. In particular, you will explore the definition of an inverse function, what types of functions have inverses, and some language to help you talk about inverse functions.

1. Let $f(x) = \dfrac{2}{x-1}$ and $g(x) = \dfrac{2}{x} + 1$.

   (a) Describe $f(x)$ in terms of function transformations of the toolkit function $\dfrac{1}{x}$. Remember that order matters!

   (b) Describe $g(x)$ in terms of function transformations of the toolkit function $\dfrac{1}{x}$. Remember that order matters!

   (c) Compute $(f \circ g)(x)$.

   (d) Compute $(g \circ f)(x)$.

   (e) How can you connect your answers to the different parts of this question? In other words, how do the transformations relate to the outcomes of the compositions?

2. What other pairs of functions f and g exist such that $f \circ g$ and $g \circ f$ are the identity function (or as I like to call it, the "do nothing function")?

   (a) What about $f(x) = -3x + 4$? Is there a $g(x)$ that works?

   (b) What about $f(x) = \frac{1}{2}(x-5)^3$? Is there a $g(x)$ that works?

   (c) What about $f(x) = x^2$? Is there a $g(x)$ that works?

   (d) Make a conjecture about which types of functions "work" in this context.

Definition: Let $f(x)$ be a function. The function $g(x)$ is an inverse of $f(x)$ if $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$ (and no domain restrictions must be imposted). We call the inverse function f^{-1}.

General Remarks on Inverse Functions:

- Only ________________ functions can have inverses.

- All ________________ functions have inverses but they may be *wildy difficult* to express in a formula.

- Computationally, the inverse of a function takes the input/output pairs of the original function and ____.

- In *every* computation of an inverse using a formula, you *must* start with the line ________________.

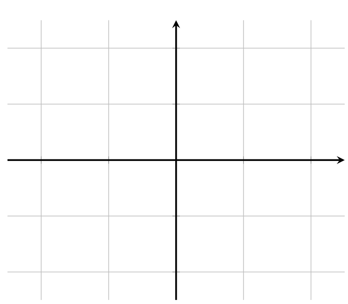
Practice:

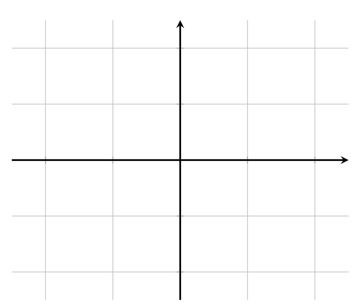
1. For the function $f(x)$ given in the table below, fill out the table for $f^{-1}(x)$.

x	$f(x)$
1	3
2	-1
3	4
4	5
5	-2

x	$f^{-1}(x)$

2. Suppose you have some one-to-one function and you're given it's graph. How do you show the inverse graphically? Sketch your hypothesis below.





3. If f is one-to-one and $f(-11) = 3$, then compute $f^{-1}(3)$ and $\big(f(-11)\big)^{-1}$.

4. Let $f(x) = \dfrac{x+4}{x+6}$. Find $f^{-1}(-3)$.

5. Let $f(x) = 6x + 3$. Find $f^{-1}(x)$.